

SUPERSYMMETRIC SCATTERING IN TWO DIMENSIONS^a

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We briefly review results on two-dimensional supersymmetric quantum field theories that exhibit factorizable particle scattering. Our particular focus is on a series of $N=1$ supersymmetric theories, for which exact S -matrices have been obtained. A Thermodynamic Bethe Ansatz (TBA) analysis for these theories has confirmed the validity of the proposed S -matrices and has pointed at an interesting ‘folding’ relation with a series of $N=2$ supersymmetric theories.

1 Introduction

To many of the practitioners of experimental or theoretical High Energy Physics, the subject of integrable Quantum Field Theories (QFT) in $1+1$ (Minkowski) space-time dimensions may seem like a rather remote corner of their field. Real particles (and people, for that matter) live in four dimensions and the field theories that are of direct relevance for Particle Physics are certainly not integrable. Thus, work on QFT in $1+1$ dimensions may seem to be a rather pointless exercise.

We would like to use the space allotted to us here to argue to the contrary, and to illustrate our point by citing some recent results^{1,2} on supersymmetric particle scattering in $d=1+1$ dimensions.

One fact of life in four dimensions is that some of the QFT’s that we need to do physics, in particular Quantum Chromodynamics (QCD), cannot satisfactorily be analyzed on the basis of perturbation theory alone, and that some of the essential physics in those theories is of non-perturbative nature. We do know of certain lower dimensional QFT’s that exhibit some of those same phenomena, and that are at the same time integrable, i.e. exactly solvable. Such lower dimensional theories offer a testing ground where the results of perturbation theory together with non-perturbative techniques can be tested against exact solutions. As an example we may cite the recent studies of the two dimensional version of QCD (QCD₂), which have in particular focused on the possibility of a string theoretical formulation of some of the strong coupling effects in the QCD₄.

More direct applications of $d=1+1$ QFT become possible as soon as the essential physics takes place in the radial direction of three-dimensional space. The radial coordinate r lives on a half-line, and taking only radial

degrees of freedom into account places us directly in the context of $d=1+1$ QFT. Note that such applications require an understanding of what happens at the boundary of the half-line, i.e. at the origin of three-dimensional space. The answer to this question may become very interesting if we assume the presence of a non-trivial object, such as a magnetic monopole or a black hole, at the origin. Phenomena such as the Callan-Rubakov effect (the catalysis of baryon decay through magnetic monopoles) and the Hawking effect (the quantum radiation of black holes) may thus be analyzed using QFT’s that are essentially one-dimensional. In general, the treatment of non-trivial boundary interactions requires some care, as the bulk QFT may be affected in a rather drastic way. In recent years, techniques to analyze boundary theories (both conformal and massive) have been developed and notions such as integrability have successfully been extended to the situation with boundary.

Yet another way in which 2d QFT is of relevance is of course in string theory, which is largely formulated as a $d=2$ QFT on a dynamical (world-sheet) surface.

Last but not least we would like to stress the fact that the subject of $d=1+1$ (integrable) QFT is one of those areas where theoretical physicists and mathematicians with entirely different backgrounds come together and share ideas. Two-dimensional field theories are of great interest for Statistical Mechanics and Condensed Matter Theory. For example, the aforementioned boundary theories have found beautiful applications in the analysis of the (multi-channel) Kondo problem and of problems involving the tunneling of edge currents in Fractional Quantum Hall devices. Thus, as it has often happened in the history of Physics, theorists working on entirely different physical problems are sharing the same tools and formalisms. Once recognized, this insight (which tends to be ignored by many of our funding and publishing agencies) can be used to the benefit of all involved.

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2 Supersymmetry in two dimensions

Supersymmetric extensions of 1+1 dimensional QFT's are entirely natural from the point of view of supersymmetric particle theories in four dimensions and superstrings. Superstrings may be viewed as world sheet field theories with superconformal invariance, i.e. with symmetries that form a supersymmetric extension of the conformal (Virasoro) algebra. In general, superconformal field theories may be perturbed by relevant perturbations that respect the supersymmetry, and this then leads to massive supersymmetric theories. For well-chosen perturbations the resulting massive theory may be integrable, by which we mean that it possesses an infinite number of non-trivial integrals of motion beyond energy and momentum.

Around 1990, many examples of integrable supersymmetric massive particle theories in $d=1+1$ dimensions have been studied, the majority possessing either $N=1$ or $N=2$ supersymmetry. These theories are typically characterized by an explicit lagrangian or, as outlined above, as a perturbation of a superconformal field theory. One of the great challenges is to determine the exact many-particle scattering amplitudes, i.e. the S -matrix. For this problem the property of integrability turns out to be of great help, since that implies the *factorizability* of the S -matrix, meaning that particle production is entirely forbidden and that all N -particle amplitudes are simply obtained as products of 2-particle scattering matrices. The consistency of such a decomposition requires a special property of the 2-particle S -matrices, the so-called Yang-Baxter equation. Additional consistency conditions arise from the compatibility of the higher integrals of motion with the bound-state structure of the S -matrix. Together these conditions are so restrictive that one may approach the problem of finding an exact S -matrix by what is called a 'bootstrap approach': one uses the consistency conditions to construct a set of 'minimal' consistent S -matrices and then tries to match those with the theories at hand.

One way to test the conjectured identification of an exact S -matrix is through the so-called Thermodynamic Bethe Ansatz (TBA), which is a procedure that allows one to derive thermodynamical properties of field theories specified by a (factorizable) S -matrix. Some of these thermodynamic quantities, such as the central charge and the scaling dimensions of the ultraviolet limit of the theory, are known from the start and can thus be used as a non-trivial consistency check on a proposed S -matrix.

In a 1990 paper¹, one of us considered the simplest type of supersymmetry in 1+1 dimensions, which is characterized by a 1+1 dimensional super-Poincaré algebra without central charges

$$Q^2 = P, \quad \overline{Q}^2 = \overline{P}, \quad \{Q, \overline{Q}\} = 0. \quad (1)$$

The paper¹ proposed a set of exact scattering matrices of the form

$$S = S_B^{[ij]} S_{BF}^{[ij]}, \quad i, j = 1, 2, \dots, n, \quad (2)$$

where $S_B^{[ij]}$ is a diagonal S -matrix for a collection of bosonic particles $\{b_1, b_2, \dots, b_n\}$, of masses m_1, m_2, \dots, m_n , and $S_{BF}^{[ij]}$ is a universal 'Bose-Fermi' S -matrix that describes the mixing of these bosonic particles with the corresponding fermions $\{f_1, f_2, \dots, f_n\}$. In words, the formula (2) describes a theory that is a supersymmetrization of a bosonic theory with diagonal scattering. Notice that the full S -matrix (2) is non-diagonal.

The paper [1] showed that supersymmetry and factorizability alone determine the form of the Bose-Fermi S -matrix $S_{BF}^{[ij]}$ up to one free constant α . Furthermore, it derived a condition for the consistency of the full S -matrix, stating that as soon as the particle b_k may occur (according to the bosonic factor $S_B^{[ij]}$ of the S -matrix) as a bound state of particles b_i and b_j , the following relation should hold

$$\alpha = - \frac{(2m_i^2 m_j^2 + 2m_i^2 m_k^2 + 2m_j^2 m_k^2 - m_i^4 - m_j^4 - m_k^4)^{\frac{1}{2}}}{2m_i m_j m_k}. \quad (3)$$

Note that this condition is extremely restrictive, since one single free parameter α should be adjusted to accommodate a large number of non-zero three-point couplings. The three-point couplings f_{ijk} of the bosonic theory branch into couplings $f_{f_i f_j b_k}$, $f_{f_i b_j f_k}$, $f_{b_i f_j f_k}$ and $f_{b_i b_j b_k}$, whose ratios depend only on the masses of the particles involved, according to

$$\frac{f_{f_i f_j b_k}}{f_{b_i b_j b_k}} = \left(\frac{m_i + m_j - m_k}{m_i + m_j + m_k} \right)^{\frac{1}{2}}. \quad (4)$$

In ref. [1] it was proposed that a specific series of perturbed $N=1$ superconformal field theories lead to a scattering theory that is precisely of the above type. The superconformal field theories are the non-unitary minimal superconformal models labeled as $M(4n+4, 2)$, of central charge

$$c_n = - \frac{3n(4n+3)}{2(n+1)}, \quad (5)$$

and the perturbing operator is the bottom component of the Neveu-Schwarz superfield labeled as $\phi_{(1,3)}$. The resulting massive theories contain n $N=1$ multiplets (b_i, f_i) of mass

$$m_i = \frac{\sin(i\beta\pi)}{\sin(\beta\pi)}, \quad \beta = \frac{1}{2n+1}, \quad (6)$$

and the parameter α takes the value

$$\alpha = \alpha_n = -\sin(\beta\pi). \quad (7)$$

The bosonic factor S_B of the S -matrix is identical to the S -matrix first described in [3], and the corresponding bound state structure is precisely such that the criterion (3) is satisfied for all non-vanishing couplings.

3 TBA and the relation with $N=2$

The Thermodynamic Bethe Ansatz (TBA) for the series of $N=1$ supersymmetric S -matrices has recently been completed², see also [4]. At the technical level, this has been somewhat of a tour de force due to the fact that the full $N=1$ S -matrix is non-diagonal. This implies that the fundamental TBA equation for the rapidities of L particles that live on a circle contains a matrix of dimension $2^L \times 2^L$, and in order to complete the TBA the eigenvalues of this matrix have to be determined. After re-interpreting the 2-particle S -matrices as Boltzmann weights, one may recognize the transfermatrix as coming from an eight-vertex model. For the explicit evaluation of these eigenvalues we have, following [5,4] relied on the fact that the Boltzmann weights satisfy what is called a ‘free fermion condition’, which allows one to avoid the generalized Bethe Ansatz that is needed for the general eight-vertex model.

The results of the TBA analysis have confirmed, among other things, the value (5) of the ultraviolet central charge and, thereby, the validity of the proposed S -matrices for the perturbed superconformal theories. We refer to our paper² for the details of this analysis.

With the TBA for the $N=1$ theory completed, we are in a position to compare a number of theories^{6,7,1} that all have the mass spectrum given by (6) and that have essentially the same fusion rules,

1. $2n$ bosonic particles $b_i, \bar{b}_i, i = 1, 2, \dots, n$, perturbation of CFT of central charge $c = \frac{4n}{2n+3}$,
2. n bosonic particles $b_i, i = 1, 2, \dots, n$, perturbation of CFT of central charge $c = -\frac{2n(6n+5)}{2n+3}$,
3. $4n$ particles $b_i, \bar{b}_i, f_i, \bar{f}_i, i = 1, 2, \dots, n, N=2$ supersymmetric perturbation of $N=2$ CFT of central charge $c = \frac{3n}{n+1}$,
4. $2n$ particles $b_i, f_i, i = 1, 2, \dots, n, N=1$ supersymmetric perturbation of $N=1$ CFT of central charge $c = -\frac{3n(4n+3)}{2(n+1)}$.

The mass spectrum and fusion rules that these theories have in common are clearly linked to the Lie algebra A_{2n} ; the first two theories in this list are usually denoted as the $A_{2n}^{(1)}$ and $A_{2n}^{(2)}$ scattering theories, respectively.

Let us now briefly discuss the relations among the theories in this list. Clearly, theory 3 is an $N=2$ supersymmetrization of theory 1 and, similarly, theory 4 is an $N=1$ supersymmetrization of theory 2. In addition,

theories 2 and 4 are obtained from 1 and 3 by eliminating half of the particles from the theory. Between the theories 1 and 2 there is a simple (multiplicative) relation^{8,6} at the level of the S -matrices, which leads to an additive relation at the level of the TBA equations. This relation has been called ‘folding in half’. For the TBA systems of the $N=2$ and $N=1$ supersymmetric theories (theories 3 and 4) a similar ‘folding’ relation was conjectured by E. Melzer⁹. Our explicit computation of the $N=1$ TBA equations have confirmed this conjecture and established the folding relation. It is expected that this folding can be traced back to a folding relation at the level of the $N=2$ and $N=1$ S -matrices, but this has not been worked out. [Writing folding relations at the S -matrix level is not straightforward since the Yang-Baxter equation, which is automatic in the bosonic case, poses severe restrictions in the supersymmetric version.]

Before closing, we would like to stress that the entire structure discussed here can be extended to $N=1$ supersymmetric boundary theories. In that case the S -matrix data are supplemented by a boundary reflection matrix R , which is subject to a boundary Yang-Baxter equation. A publication on these matters is in preparation¹⁰.

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